Topics in Mathematics of Information
(A very brief overview)

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Admin

- Webpage: `www.damtp.cam.ac.uk/user/cmhsp2/teaching.html`
- There will be 3 example classes. First class in the week beginning 23rd October. Details to follow.
- There will be a 3 hour exam in June.

This course

We are surrounded by technology that collects, transmits, process and interpret reams of data.

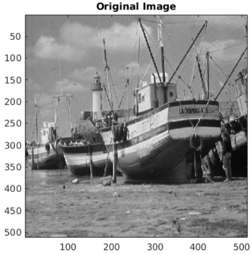
In this course, we shall cover some topics concerning:

1. How can we represent images/signals efficiently?
2. How do we exploit the structure of the underlying data to extract meaningful information?

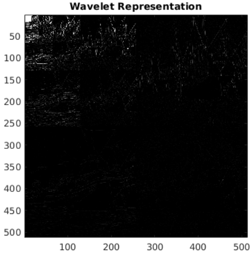
I: Discrete representations

An important reason why we are able to handle huge volumes of data is that that the data of interest is not random – it is inherently structured.

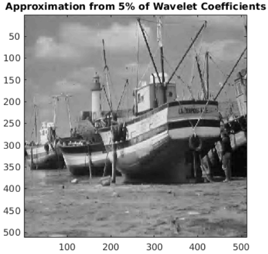
In particular, much of the data is sparse with respect to some representation system and can be compressed. Many of these representation systems are closely linked to wavelet bases.



I



$\{\langle I, \psi_{j,k}\rangle : k, j \in \mathbb{Z}\}$



$\sum_{k,j\in\Lambda} \langle I, \psi_{j,k}\rangle \psi_{j,k}$

II: Compressed sensing

Candés, Romberg & Tao (2006); Donoho (2006)

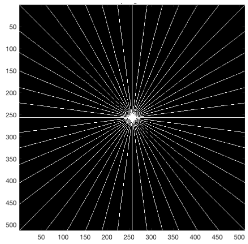
Task: Given $y_0 = Ax_0$ where $A : \mathbb{R}^N \to \mathbb{R}^m$ with $N \gg m$, recover x_0.

This is impossible in general, since we have more unknowns than knowns.

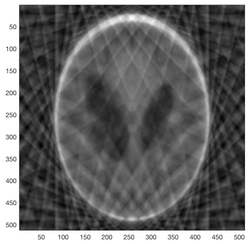
However, the key observation of compressed sensing is that one can exploit the fact that natural images have sparse representations.

Take the reconstruction $\hat{x}$ as

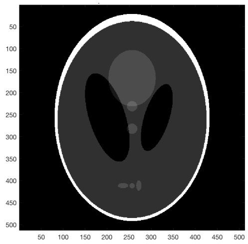
$$\operatorname*{argmin}_x \|Wx\|_0 \text{ subject to } Ax = y_0$$



Sampling coefficients



Direct Fourier inversion

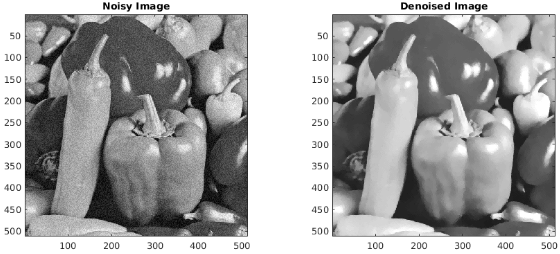


Sparse reconstruction

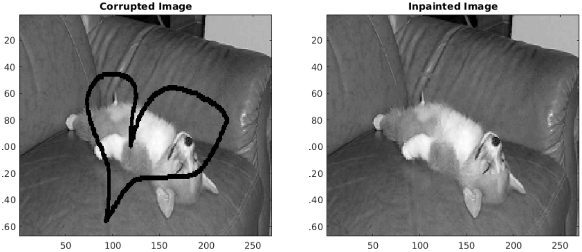
III: Exploiting geometric structure in inverse problems

More generally, one can deal with ill-posed inverse problems by considering the geometric structures of the underlying objects, such as edges and curvature.

Denoising:



Inpainting:



Wavelet representations: idea

218,228,215,223,221,225,226,127,106,106, 132,132,129,130,129,128.

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$$\begin{cases} \textit{Averages} : 223, 219, 223, 176.5, 106, 132, 129.5, 128.5. \\ \end{cases}$$

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$$\begin{cases} \textit{Averages} : 223, 219, 223, 176.5, 106, 132, 129.5, 128.5. \\ \textit{Differences} : 10, 8, 4, -99, 0, 0, 1, -1, \end{cases}$$

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$$\begin{cases} A : 221, 199.75, 119, 129, \\ D : -4, -46.5, 26, -1, \end{cases}$$

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Wavelet decomposition

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